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ON THE EUCLIDEAN CONNECTION IN AN AREAL SPACE OF GENERAL TYPE (II)

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矢野晋平： 面積空間における Euclidean Connection について

§ 1. Introduction.

In the previous paper [8] [1], we studied the Euclidean connection in an areal space endowed with *two*-dimensional area, i.e. m = 2. In the present paper as the generalization of the previous case, we shall study the theories with a general case, i.e. $1 \leqq m \leqq n-1$.

We define the partial derivatives of a homogeneous function Φ of degree ρ in p^i_α as

$$(1.1) \qquad \Phi_{;i[m]} \equiv m\Phi_{,[i_1}^{1} p^2_{i_2} \cdots p^{m]}_{i_m]} - (m-1)\rho\Phi p_{i[m]} \ ^{2)},$$

where

$$(1.2) \qquad F_{,i}^{\alpha} p^i_\beta = \delta^\alpha_\beta F, \qquad p^\alpha_i \equiv F^{-1} F_{,i}^{\alpha},$$

$$(1.3) \qquad p_{i[m]} \equiv p^{[1}_{i_1} \cdots p^{m]}_{i_m}, \qquad p^{i[m]} \equiv m!\, p^{i_1}_{[1} \cdots p^{i_m}_{m]}.$$

The partial derivatives $\Phi_{;i[m]}$ were employed in the case of $\rho = 0$ or another forms of (1.1) by several authors [2]—[5].

In § 2 we show that $\Phi_{;i[m]}$ is homogeneous of degree $\rho - 1$ in p^i_α, in consequence of this property the second partial derivatives $\Phi_{;i[m];j[m]}$ can be introduced and these functions are not of symmetry with i and j. The components of metric m-tensor $g_{i[m],j[m]}$ are represented by algebraic functions of $L_{;i[m];j[m]}$, $p^{i[m]}$ and $p_{i[m]}$ in § 3. We define the Euclidean connection as $Dg_{i[m],j[m]} = 0$ then the connection parameters $C_i{}^j{}_{k[m]}$ and $\Gamma^{*i}_{j\ k}$ are determined unequally by the fundamental function and its derivatives under convenient conditions in § 4. The three kind of curvature tensors $R_i{}^j{}_{kl}$, $P_i{}^j{}_{k[m]l}$ and $S_i{}^j{}_{k[m]l[m]}$ are obtained in § 5.

§ 2. The partial derivatives.

Let us consider the function $\Phi(x, p^i_\alpha)$ which satisfies relations

1) Number in bracket refers to the references at the end of the paper.

2) We take the notations $F^\alpha_i \equiv \partial F/\partial p^i_\alpha$, $\Phi_{,i}^{\alpha} \equiv \partial\Phi/\partial p^i_\alpha$, $\Phi_{,i,j}^{\alpha\beta} \equiv \partial^2\Phi/\partial p^i_\alpha \partial p^j_\beta$ throughout present paper, where Latin indices i, j, k, l, etc. run over $1, 2, \cdots, n$, but Greek indices $\alpha, \beta, \gamma, \delta$, etc., over $1, 2, \cdots, m$, $n > m$.

$$(2.1) \qquad \Phi_i^{\alpha}\, p_\beta^i = \rho\Phi\delta_\beta^\alpha.$$

After not so complicated calculations we obtain $mm!\,\Phi_i^{[1}\, dp_{[1}^{i]}\,\delta_2^2 \cdots\cdots \delta_{m]}^{m]} = \Phi_{,i}^{\alpha}\, dp_\alpha^i = d\Phi$, from which we can get without difficulty

Theorem [2.1]. *If a function* $\Phi(x, p_\alpha^i)$ *is homogeneous of degree* ρ *in* p_α^i, *the partial derivatives satisfy the relations*

$$(2.2) \qquad \Phi_{;i[m]}\, dp^{i[m]} = d\Phi, \qquad\qquad \Phi_{;i[m]}\, p^{i[m]} = \rho\Phi^{1)}.$$

Differentiating both sides of (1.2) with respect to p_γ^j we have

$$(2.3) \qquad F_{,j,i}^{\gamma\,\alpha}\, p_\beta^i = 2F_{,j}^{[\gamma}\,\delta_\beta^{\alpha]}, \qquad p_{i,j}^{\alpha\,\beta} = -p_j^\beta p_i^\alpha + F^{-1}F_{,i,j}^{\alpha\,\beta}, \qquad p_{i,j}^{\alpha\,\beta}\, p_\gamma^j = -p_i^\beta\,\delta_\gamma^\alpha,$$

consequently we obtain $p_{i[m],j}^{\quad\alpha}\, p_\beta^j = -p_{i[m]}\,\delta_\beta^\alpha$, these equations tell us

Theorem [2.2]. $p_{i[m]}$ *is homogeneous of degree* -1 *in* p_α^i.

Differentiating both sides of (2.1) with respect to p_γ^j we obtain $\Phi_{,j,i}^{\gamma\,\alpha} p_\beta^i = \rho\Phi_{,j}^{\gamma}\delta_\beta^\alpha - \Phi_{,j}^{\alpha}\,\delta_\beta^\gamma$, from which we have on using the second relations of (2.3)

$$(2.4) \qquad (\partial\Phi_{,[i_1}^{[1}\, p_{i_2}^2 \cdots p_{i_m]}^{m]}/\partial p_\alpha^j)\, p_\beta^j = (\rho - 1)\delta_\beta^\alpha\, \Phi_{[i_1}^{[1}\, p_{i_2}^2 \cdots p_{i_m]}^{m]},$$

(2.4) tells us that $\Phi_{,[i_1}^{[1}\, p_{i_2}^2 \cdots p_{i_m]}^{m]}$ is homogeneous of degree $(\rho - 1)$ in p_β^j, consequently by means of Theorem [2.2] we have

Theorem [2.3]. *If* Φ *is a function of homogeneous of degree* ρ *in* p_α^i, *then* $\Phi_{;i[m]}$ *is homogeneous of degee* $\rho - 1$ *in* p_α^i, *i.e.*

$$(2.5) \qquad (\partial\Phi_{;i[m]}/\partial p_\alpha^j)\, p_\beta^j = (\rho - 1)\,\Phi_{;i[m]}\,\delta_\beta^\alpha.$$

In virtue of Theorems [2.2] and [2.3] we can define the second and third partial derivatives of Φ, and from Theorem [2.1] there are relations among them

$$(2.6) \qquad \Phi_{;i[m];j[m]}\, p^{j[m]} = (\rho - 1)\,\Phi_{;i[m]}, \qquad \Phi_{;i[m];j[m];k[m]}\, p^{k[m]} = (\rho - 2)\,\Phi_{;i[m];j[m]}.$$

After not so complicated calculations we obtain on using of the second relations of (2.3) and (1.1)

$$\begin{aligned}(2.7) \quad \Phi_{;i[m];j[m]} &= m\Phi_{;i[m],[j_1}^{\qquad [1}\, p_{j_2}^2 \cdots p_{j_m]}^{m]} - (m-1)(\rho-1)\,\Phi_{;i[m]}\, p_{j[m]} \\ &= m^2\Phi_{,[i_1,[j_1}^{[1\ \ [1}\, p_{i_2}^2 \cdots p_{i_m]}^{m]}\, p_{j_2}^2 \cdots p_{j_m]}^{m]} - m(m-1)(m+\rho-1)\,\Phi_{,[i_1}^{[1}\, p_{i_2}^2 \cdots p_{i_m]}^{m]}\, p_{j[m]} \\ &\quad - m(m-1)\rho\,\Phi_{,[j_1}^{[1}\, p_{j_2}^2 \cdots p_{j_m]}^{m]}\, p_{i[m]} + \rho\Phi(m-1)(m^2 + m\rho - m - \rho + 1)\, p_{i[m]}\, p_{j[m]} \\ &\quad + m^2(m-1)F^{-1}\Phi_{,[i_1}^{[1}\, p_{i_2}^2 \cdots p_{i_{m-1}}^{m-1}\, F_{,i_m],[j_1}^{m]\ \ [1}\, p_{j_2}^2 \cdots p_{j_m]}^{m]} \\ &\quad - m^2(m-1)\rho\Phi F^{-1} F_{,[i_1,[j_1}^{[1\ \ [1}\, p_{i_2}^2\, p_{i_m]}^{m]}\, p_{j_2}^2 \cdots p_{j_m]}^{m]},\end{aligned}$$

from which we can get without difficulty by means of (2.3)

$$(2.8) \qquad \Phi_{;i[m];j[m]} - \Phi_{;j[m];i[m]} = (m-1)\{\Phi_{;i[m]}\, p_{j[m]} + m^2\Phi_{,[i_1}^{[1}\, p_{i_2}^2 \cdots p_{i_m],[j_1}^{m]\ [1}\, p_{j_2}^2 \cdots p_{j_m]}^{m]}\} - cycl\,(ij).$$

From (2.8) we have

1) See [5] Theorem [1.6].

Theorem [2.4]. *An areal space is Finsler or Riemannian one, when and only when the partial differential operation with any function is commutative for the order.*

Multiplying the both sides of (2.8) $p^{i[m]}$ and summing for i, from (2.3) we have

$$(2.9) \qquad (\Phi_{;i[m];j[m]} - \Phi_{;j[m];i[m]})p^{i[m]} = 0.$$

§ 3. Metric m-tensor and unit m-vector.

If we put L in (2.2), (2.6), (2.7) and (2.9) instead of Φ, because of the fundamental function L is homogeneous of degree two in p^i_α, we have

$$(3.1) \quad L_{;i[m];j[m]} = L_{;j[m];i[m]} = 2L(1+m-m^2)p_{i[m]}p_{j[m]} + m^2 F F_{,[i_1}^{[1}{}_{,[j_1}^{[1} p^2_{i_2} \cdots p^{m]}_{i_m]} p^2_{j_2} \cdots p^{m]}_{j_m]},$$

$$(3.2) \qquad L_{;i[m];j[m]} p^{i[m]} p^{j[m]} = 2L,$$

$$(3.3) \qquad L_{;i[m];j[m];k[m]} p^{j[m]} = 0.$$

(3.1), (3.2) and (3.3) show that $L_{;i[m];j[m]}$ satisfies Iwamoto's relations[1)] which are satisfied by the metric m-tensor $g_{i[m],j[m]}$[2)], but we can not take $L_{;i[m];j[m]}$ for metric m-tensor instead of $g_{i[m],j[m]}$. However we can express $g_{i[m],j[m]}$ by function of $L_{;i[m];j[m]}$ and $p^{i[m]}$.

Let $\bar{F}^{\alpha\beta}_{ij}$ be Legendre's form of F, i.e., $\bar{F}^{\alpha\beta}_{ij} = F^{-1}F_{,i}^{\alpha}{}_{,j}^{\beta} - p^\alpha_i p^\beta_j + p^\beta_i p^\alpha_j$, after some calculations we have

$$(3.4) \qquad \bar{F}^{\alpha\beta}_{ij} = X^{\alpha\beta}_{ij} - p^\alpha_i p^\beta_j,$$

where $\quad X^{\alpha}_{i_\alpha}{}^{\beta}_{j_\beta} \equiv F^{-2}(m!)^2 L_{;i[m];j[m]} p^{i1}_1 \cdots p^{i\alpha-1}_{\alpha-1} p^{i\alpha+1}_{\alpha+1} \cdots p^{im}_m p^{j1}_1 \cdots p^{j\beta-1}_{\beta-1} p^{j\beta+1}_{\beta+1} \cdots p^{jm}_m.$

From (3.4) the metric m-tensor $g_{i[m],j[m]}$ can be represented by $X^{\alpha\beta}_{ij}$, for examle in the case of $m=2$, we have

$$(3.5) \qquad g_{ij,kl} = L(-2p_{ij}p_{kl} + 2p^{[1}_{[i} p^{[1}_{[k} X^{2]2]}_{j]l]} + X^{[1[1}_{[i[k} X^{2]2]}_{j]l]}).$$

The contravariant components $g^{i[m],k[m]}$ of the metric m-tensor $g_{i[m],j[m]}$ are deduced by the relations $g_{i[m],j[m]} g^{i[m],k[m]} = (m!)^2 \delta^{[k_1}_{j_1} \cdots \delta^{km]}_{jm}$. If we define unit contravariant and covariant m-vectors respectively as the relations $l^{j[m]} \equiv F^{-1}p^{i[m]}$, $l_{i[m]} \equiv F^{-1}L_{;i[m]}$, there are the relations among them $g_{i[m],j[m]} l^{i[m]} l^{j[m]} = 1$, $g_{i[m],j[m]} l^{i[m]} = l_{j[m]}$.

§ 4. Euclidean Connection Parameters.

We shall determine the connection parameters according to the way taken in Finsler and Cartan spaces.

At first we show the relations used in following calculations : Let Ψ^i_j be a function providing two components i and j, then we have without difficulty

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1) See [7], [4] (9.1), (9.2).

2) See [7], [1] (3.16), [3] (1.14).

$$(4.1)\qquad m!\,\delta^{[i_1}_{i_1}\,\delta^{i_2}_{i_2}\,\delta^{i_3}_{i_3}\cdots\delta^{i_m]}_{i_m}=n(n-1)\cdots(n-m+1),$$

$$(4.2)\qquad m!\,\delta^{[k_1}_{j_1}\,\delta^{i_2}_{i_2}\cdots\delta^{i_m]}_{i_m}=\delta^{k_1}_{j_1}(n-1)(n-2)\cdots(n-m+1),$$

$$(4.3)\qquad m!\,\Psi^{[k_1}_{j_1}\,\delta^{i_2}_{i_2}\cdots\delta^{i_m]}_{i_m}=\Psi^{k_1}_{j_1}(n-1)(n-2)\cdots(n-m+1),$$

$$(4.4)\qquad m!\,\Psi^{[i_2}_{i_2}\,\delta^{k_1}_{i_1}\,\delta^{i_3}_{i_3}\cdots\delta^{i_m]}_{i_m}=(n-2)(n-3)\cdots(n-m+1)(\Psi^{i_2}_{i_2}\,\delta^{k_1}_{i_1}-\Psi^{k_1}_{i_1}).$$

Let us consider an intrinsic covariant vector X_i and introduce into this space an absolute derivation for the vector in the form

$$(4.5)\qquad DX_i=dX_i-\Gamma_i{}^b{}_a\,X_b\,dx^a-C_i{}^a{}_{j[m]}\,X_a\,dp^{i[m]}.$$

From the first relation of (2.2) we can deformed (4.5)

$$DX_i=(\partial_a X_i-\Gamma_i{}^b{}_a X_b)\,dx^a+(X_{i;j[m]}-C_i{}^a{}_{j[m]}X_a)dp^{j[m]}.$$

If we define the Euclidean connection as $Dg_{i[m],j[m]}=0$, then we have

$$(4.6)\qquad \partial_a g_{i[m],j[m]}=\sum_\rho(\Gamma_{i_\rho}{}^b{}_a\,g_{i[m](b),j[m]}+\Gamma_{j_\rho}{}^b{}_a\,g_{i[m],j[m](b)}),$$

$$(4.7)\qquad g_{i[m],j[m];l[m]}=\sum_\rho(C_{i_\rho}{}^b{}_{l[m]}\,g_{i[m](b),j[m]}+C_{j_\rho}{}^b{}_{l[m]}\,g_{i[m],j[m](b)}).$$

In order to derive the connection parameters $C_i{}^a{}_{j[m]}$ from the fundamental function and its derivatives in the considered domain, we assume that following relations which corespond, in Finsler and Cartan spaces, to $C_{(ij)k}=0$, are satisfied in the domain

$$(4.8)\qquad \sum_\rho C_{i_\rho}{}^b{}_{k[m]}\,g_{i[m](b),j[m]}=\sum_\rho C_{j_\rho}{}^b{}_{k[m]}\,g_{i[m],j[m](b)}.$$

Multiplying both sides of (4.7) $g^{k[m],j[m]}$ and summing for $_{j[m]}$ we have

$$(4.9)\qquad g_{i[m],j[m];l[m]}\,g^{k[m],j[m]}=2(m!)^2\sum_{\rho=1}^{m}C_{i_\rho}{}^b{}_{l[m]}\,\delta^{[k_1\cdots k_m]}_{i[m](b)}.$$

Putting $k_2=i_2,\cdots,k_m=i_m$ on above equations and summing for same indices making us of (4.3) and (4.4) we have

$$(4.10)\qquad C_b{}^a{}_{l[m]}=\{2(m!)(n-1)\cdots(n-m)\}^{-1}\{(n-1)g_{bi_2\cdots i_m,j[m];l[m]}g^{ai_2\cdots i_m,j[m]}$$
$$-m^{-1}\delta^a_b\,g_{i[m],j[m];l[m]}g^{i[m],j[m]}\}$$

Since $g_{i[m],j[m]}$ is homogeneous of degree zero in p^i_α, if we put $A_b{}^a{}_{l[m]}\equiv F^{-1}C_b{}^a{}_{l[m]}$ then $A_b{}^a{}_{l[m]}$ may be homogeneous of degree zaro and from (4.10) satisfies the relations

$$(4.11)\qquad A_b{}^a{}_{l[m]}\,p^{l[m]}=0.$$

Because of Theorem [2.1] and (4.11) relation (4.5) reduce to

$$(4.12)\qquad DX_i=(\partial_a X_i-\Gamma_i{}^b{}_a X_b)dx^a+(X_{i\|j[m]}-A_i{}^a{}_{j[m]}X_a)dl^{j[m]},$$

where $X_{i\|j[m]}\equiv F^{-1}X_{i;j[m]}$. Multiplying (4.9) $p^{j[m]}$ and summing with respect to $_{i[m]}$ considering the relations $g_{i[m],j[m];k[m]}\,p^{i[m]}=0$[1], we have

1) Since $g_{i[m],j[m]}$ is homogeneous of degree zero in p^i_α, $g_{i[m],j[m];k[m]}$ coincides with partial derivativ of $g_{i[m],j[m]}$ in sense of definition of A. KAWAGUCHI, See [3] (1.9), [4] (9.2).

(4. 13) $\sum_{\rho} C_{i\rho}{}^{k\rho}{}_{l[m]}\, p k_1 k_2 \cdots k_{\rho-1} k_{\rho+1} \cdots k_m = 0.$

From above equations we can determine the base connection

(4. 14) $Dl^{i[m]} = dl^{i[m]} + \Gamma_0{}^{i[m]}{}_a\, dx^a,$ where $\Gamma_0^{i[m]}{}_\alpha = \sum_{\rho} \Gamma_a{}^{i\rho}{}_\alpha\, l i_1 \cdots i_{\rho-1} a i_{\rho+1} \cdots i_m.$

From (4.14) relations of (4.12) are reduced to

(4. 15) $DX_i = X_{i|a}\, dx^a + X_{i||j[m]}\, Dl^{j[m]},$

where $X_{i|a} \equiv \partial_a X_i - \Gamma^{*b}_{i\ a} X_b - X_{i||j[m]}\, \Gamma_0{}^{j[m]}{}_a,$ $\qquad X_{i||j[m]} \equiv X_{i,|j[m]} - A_i{}^a{}_{j[m]}\, X_a,$

(4. 16) $\Gamma^{*b}_{i\ a} \equiv \Gamma_i{}^b{}_a - A_i{}^b{}_{j[m]}\, \Gamma_0^{j[m]}{}_a.$

$X_{i|a}$ and $X_{i||j[m]}$ are the covariant derivatives of the vector X_i in sense of the Euclidean connection. From (4.13) we have

(4. 17) $\Gamma^{*i[m]}_{0\ \ a} = \Gamma_0{}^{i[m]}{}_a.$

In order to derive the parameters $\Gamma_i^{*j}{}_k$ from fundamental functions, we introduce the conditions which are supposed in Finsler and Cartan spaces

(4. 18) $\Gamma^{*l}_{j\ k} = \Gamma^{*l}_{k\ j}.$

We must solve (4.6) for $\Gamma_i^{*j}{}_k$ under above conditions, but this purpose is complicate, so we confine ourselves in a case $m=2$.

If the space is regular, we have the metric tensor with 4-index $A_{ij,kl} \equiv (n-1)^{-1}(n-2)^{-1}\,(g_{i\gamma,j)s}\, g^{p\gamma,qs}\, g_{p(k,q|l)} - g_{(i(k,j)l)})$ which is symmetric not only in the indices i and j as will as k and l but also in the pairs of indices (i,j) and (k,l) and degenerates to $g_{ij}\, g_{kl}$ for a metric space[1)]

Since $A_{ij,kl}$ consists of $g_{ab,cd}$ and $g^{ab,cd}$, so we have $DA_{ij,kl} = 0$, i. e.,

(4. 19) $\partial_a A_{ijkl} = \Gamma^{*b}_{i\ a} A_{bjkl} + \Gamma^{*b}_{j\ a} A_{ibkl} + \Gamma^{*b}_{k\ a} A_{ijbl} + \Gamma^{*b}_{l\ a} A_{ijkb} + A_{ijkl||dc}\, \Gamma^{*d}_{b\ a}\, l^{bc}.$

If we put

(4. 20) $\gamma_k{}^i{}_j = \frac{1}{4} A^{abci}(\partial_k A_{acjb} + \partial_j A_{ackb} - \partial_c A_{ak,jb}),$

then $\gamma_k{}^i{}_j$ is symmetric in the indices k and j and for a metric areal space coincide with the Christoffel symbol constructed of g_{ij}.

If we put (4.19) in the right hand member of (4.20) after not so complicated calculations we obtain on using of (4.18)

(4. 21) $\gamma_k{}^i{}_j = \Gamma^{*a}_{b\ c}\, W^{ibc}_{ajk},$

where $W^{ibc}_{ajk} = \{\delta^{ibc}_{ajk} + \frac{1}{2}(A^{bdei}\, A_{dea(k}\, \delta^c_{j)} - A^{debi}\, A_{a(j,k)d}) + \frac{1}{4} l^{fb}(2A^{degi}\delta^c_{(j} A_{k)deg;aj}$

$- A^{deci}\, A_{ejkd;aj})\}.$

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1) See [1], p 38, (5.19).

If we assume that the $2n^2$-rowed determinant W constructed of W^{ibc}_{ajk} with respect to the systems of indices (abc) and (ijk) does not vanish in the considered domain, the quantities $\overline{W}^{ajk}_{ilm}$ will be determied uniquely as the solutions of the system of equations ; $W^{ibc}_{ajk}\overline{W}^{ajk}_{ilm}=\delta^{abc}_{alm}$. In a Riemannian space, there exist the relations $W^{ibc}_{ajk}=\delta^{ibc}_{ajk}$ and $W=|\delta^i_a\,\delta^b_j\,\delta^c_k|=1$. Multiplying the both sides of (4.21) $\overline{W}^{ajk}_{ilm}$ and summing with respect to i, j and k we get

$$(4.22)\qquad \Gamma^{*a}_{i\ m}=\gamma^{\ i}_{j\ k}\,W^{ajk}_{ilm}.$$

§ 5. The curvature tensors.

The curvature tensors of the Euclidean connection $R_i{}^a{}_{jk}$, $P_i{}^a{}_{j(m)k}$, $S_i{}^a{}_{j(m)k(m)}$ are obtained in usual way as the coefficients of the equations which are given by the difference of two absolute derivations.

From (4.14) and (4.17) the derivation of X_i corresponding to different d_1 is expressed by

$$(5.1)\qquad d_1X_i=(\partial_\alpha X_i-X_{i||l(m)}\Gamma^{*l(m)}_{0\ \ \alpha})dx_1^{\alpha}+X_{i|l(m)}D_1l^{l(m)},$$

from which, equation (4.5) is rewritten $D_1X_i=d_1X_i-\Gamma^{*b}_{i\ a}X_bd_1x^a-A_i{}^b{}_{j(m)}X_bD_1l^{j(m)}$.
From (5.1) and (4.14) after simple calculations we have

$$(5.2)\qquad \underset{[2\ 1]}{DD}X_i=-X_c(\partial_{[b}\Gamma^{*c}_{i|a]}-\Gamma^{*c}_{i[a||l(m)}\Gamma^{*l(m)}_{0\ \ b]}-\Gamma^{*d}_{i[b}\Gamma^{*c}_{d|\,a]})d_1x^ad_2x^b$$
$$-X_c(\Gamma^{*c}_{i\ a||j(m)}-\partial_a A_i{}^c{}_{j(m)}+\Gamma^{*l(m)}_{0\ \ a}A_i{}^c{}_{j(m)||l(m)}+\Gamma^{*d}_{i\ a}A_d{}^c{}_{j(m)}$$
$$-A_i{}^d{}_{j(m)}\Gamma^{*c}_{d\ a})\underset{[2}{D}l^{j(m)}\underset{1]}{dx^a}$$
$$-X_c(A_i{}^c{}_{j(m)||l(m)}-A_i{}^d{}_{l(m)}A_d{}^c{}_{j(m)})\underset{[1}{D}l^{l(m)}\underset{2]}{D}l^{j(m)}-X_dA_i{}^d{}_{j(m)}\underset{[2\ 1]}{dD}l^{j(m)}.$$

$$(5.3)\qquad \underset{[2\ 1]}{dD}l^{l(m)}=[\sum_\rho\{(\partial_{[b}\Gamma^{*i_\rho}_{\ |k|\ a]}-\Gamma^{*i_\rho}_{k\ [a|j(m)}\Gamma^{*j(m)}_{0\ \ b]})\,l^{i_1\cdots k\cdots i_m}$$
$$-\Gamma^{*i_\rho}_{k\ [a}\Gamma^{*(i_1\cdots k\cdots i_m)}_{0\ \ b]}\}]\,\underset{[1}{dx^a}\underset{2]}{dx^b}$$
$$+\{\sum_\rho(\Gamma^{*i_\rho}_{k\ a|j(m)}l^{i_1\cdots k\cdots i_m}+\Gamma^{*i_\rho}_{j_\rho a}\delta^{i_1}_{j_1}\cdots\delta^{i_{\rho-1}}_{j_{\rho-1}}\delta^{i_{\rho+1}}_{j_{\rho+1}}\cdots\delta^{i_m}_{j_m})\}\underset{[2}{D}l^{(jm)}\underset{1]}{dx^a}.$$

From above two equations we can get without difficulty by means of (5.2) and (5.3)

$$(5.3)\qquad 2\underset{[1\ 2]}{DD}X_i=-X_c(R_i{}^c{}_{ba}+A_i{}^c{}_{,j(m)}R_0{}^{j(m)}{}_{ba})\underset{1}{dx^a}\underset{2}{dx^b}-2X_cP_i{}^c{}_{j(m)a}\underset{[2}{D}l^{j(m)}\underset{1]}{dx^a}.$$
$$-X_cS_i{}^c{}_{k(m),j(m)}\underset{2}{D}l^{k(m)}\underset{1}{D}l^{j(m)},$$

where $R_i{}^c{}_{ba}\equiv 2(\partial_{[b}\Gamma^{*c}_{|i|\ a]}+\Gamma^{*d}_{i\ [a}\Gamma^{*c}_{|d|\ b]}+\Gamma^{*c}_{i\ [b]||l(m)}\Gamma^{*l(m)}_{0\ \ a]})$,

$$P_i{}^c{}_{j(m)a}\equiv -A_i{}^c{}_{j(m),a}+\Gamma^{*c}_{i\ a||j(m)}+A_i{}^c{}_{k(m)}(\sum_\rho\Gamma^{*k_\rho}_{d\ \ a|j(m)}l^{k_1\cdots d\cdots k_m}),$$

$$S_k{}^c{}_{i(m)j(m)}=(A_k{}^c{}_{j(m)||i(m)}-A_k{}^d{}_{i(m)}A_d{}^c{}_{j(m)})-cycl(i\,j).$$

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