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Complement on "A Note on Completeness of the Space of Continuous Functions"

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坪内昭夫：「連続関数の空間の連続性について」への補遺

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Abstract

We make up for the lack of proof on the sufficient condition which appeaed in a previous paper "A Note on Completeness of the Space of Continuous Functions" (Journal of Hokkaido University of Education (Section II A) Vol. 31, No. 2, pp. 61-62.)

Proof. Conversely, suppose E satisfies 1), 2) and 3). For any system $f_\lambda(\lambda\in\Lambda)$ of non-negative continuous functions, we put $S_\alpha^{(\lambda)}=\{x|f_\lambda(x)<\alpha\}$ and $S_\alpha=\bigcup_{\lambda\in\Lambda} S_\alpha^{(\lambda)}$ $(\alpha>0)$. By the condition 1) there exists U_α as the smallest open-closed set containing E_α. We have

$$\text{(a)}\quad 0<\alpha<\beta \Rightarrow U_\alpha\subset U_\beta \qquad\qquad \text{(b)}\quad \bigcup_{\alpha>0} U_\alpha=E.$$

If we put $f_o(x)=\inf_{x\in U_\alpha}\alpha$ for any $x\in E$, then by (b) f_o is a non-negative real valued function on E.

Furthermore by (a) as $\{x|f_o(x)<\alpha\}=\bigcup_{\alpha>\beta>0} U_\beta$ and $\{x|f_o(x)\leqq\alpha\}=\bigcap_{\beta>\alpha>0} U_\beta$, we see that f_o is a continuous function on E.

Since $S_\alpha^{(\lambda)}\subset U_\alpha\subset\{x|f_o(x)\leqq\alpha\}(\lambda\in\Lambda)$, we have $f_\lambda\geqq f_o$. If $f_\lambda\geqq g\geqq 0(\lambda\in\Lambda)$ for some $g\in C(E)$, then we have $S_\alpha^{(\lambda)}\subset\{x|g(x)\leqq\alpha\}(\alpha>0)$, so that $S_\alpha\cap\{x|g(x)>\alpha\}=\emptyset$ Hence by the assumption 2) we have $U_\alpha\cap\{x|g(x)>\alpha\}=\emptyset$, and so $\{x|f_o(x)<\alpha\}\subset U_\alpha\subset\{x|g(x)\leqq\alpha\}$. Consequently we obtain $f_o\geqq g$. Therefore f_o is an infimum of $\{f_\lambda\}$.